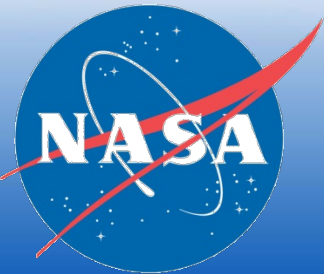


# Math Connections to Earth and Space Science



$$a^2 + b^2 = c^2$$

# Math Connections to Earth and Space Science





# *Gears and Gear Ratios*

## Introduction to Robotics

### Jan Stark




$$a^2 + b^2 = c^2$$

Math Connections to Earth and Space Science





# *Gears – What are they?*

-  Gears are wheels with teeth.
-  Gears mesh together and make things turn.
-  Gears are used to transfer **motion** or **power** from one moving part to another.



# *Gears – The Purpose*



## **POLL QUESTION**

$$a^2 + b^2 = c^2$$

Math Connections to Earth and Space Science



# Gears – The Purpose



Sports cars go fast (have speed) but cannot pull any weight.

Big trucks can pull heavy loads (have power), but cannot go fast. *Gears cause this.*

**Gears increase or decrease the power or speed, but you cannot generally speaking.**

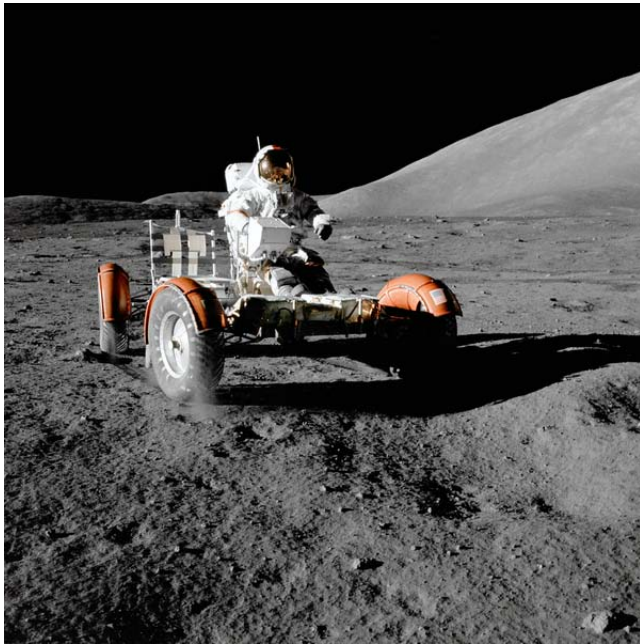




# *NASA and Gears*



Lunar Rover for Apollo  
Astronauts to explore the  
Moon's surface.



SPIRIT and Opportunity for  
the exploration of Mars



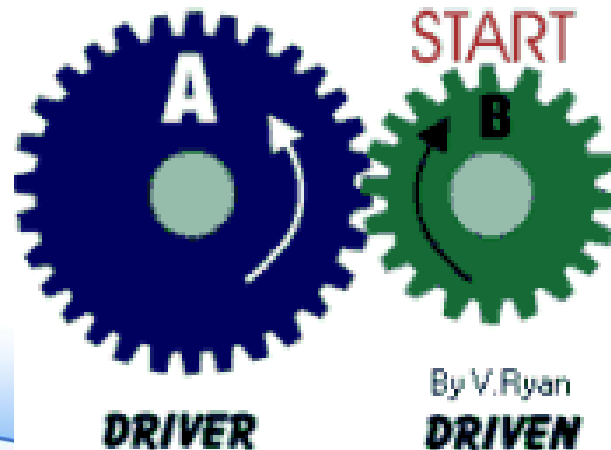
$$a^2 + b^2 = c^2$$

Math Connections to Earth and Space Science



# Types of Gears

**Spur gears** are gears in the same plane that move opposite of each other because they are meshed together. Gear 'A' is called the 'driver' because this is turned by a motor. As gear 'A' turns it meshes with gear 'B' and it turns. Gear 'B' is the 'driven' gear.



$$a^2 + b^2 = c^2$$





# Gear Details (Spur)

The circle marked in **red** shows the outer limit of the teeth. Green circles are known as the **pitch** circles.

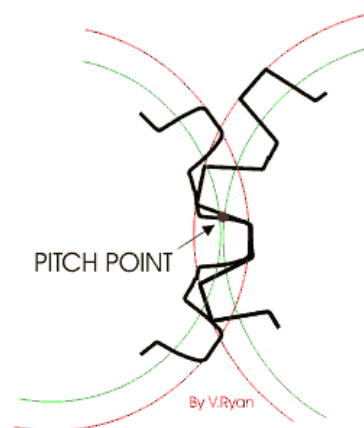
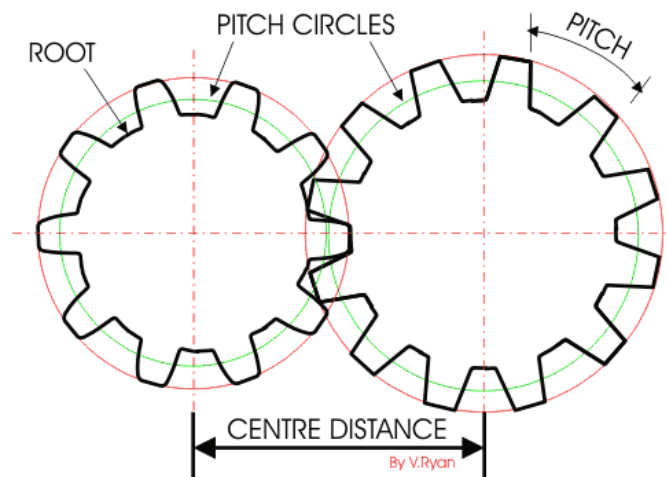
The pitch circle of a gear is used by engineers to determine:

- ⚙️ *shape* of the teeth &
- ⚙️ *ratio* between gears

(ratios will be explained later).

The **pitch** of a gear = distance between any point on one tooth and the same point on the next tooth.

The **root** is the bottom part of a gear wheel.



The **pitch point** is the point where gear teeth actually make contact with each other as they rotate.



# Types of Gears



**Bevel gears** - used to change the direction of drive in a gear system by  $90^\circ$ .

A good example is seen as the main mechanism for a hand drill.

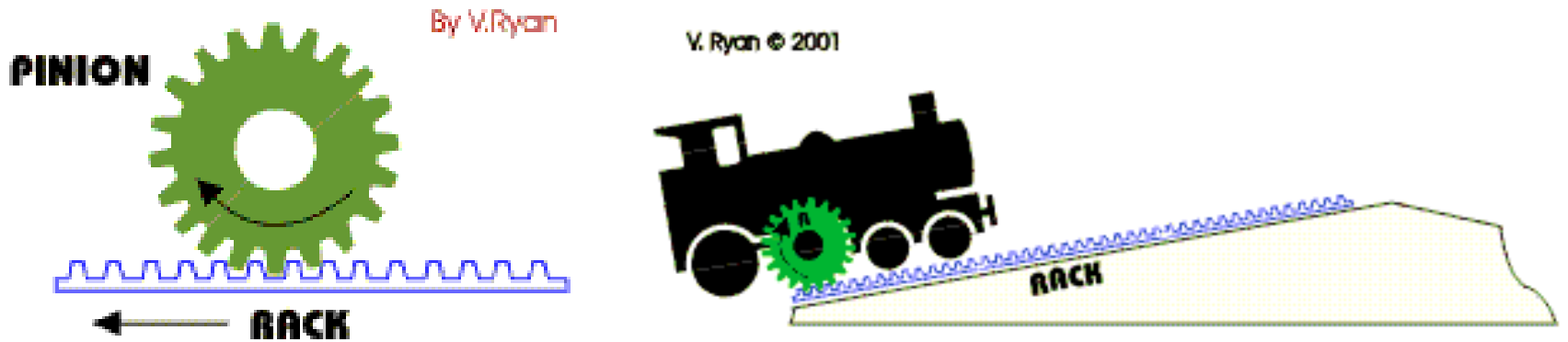
As the handle of the drill is turned in a vertical direction, the bevel gears change the rotation of the chuck to a horizontal rotation

$$a^2 + b^2 = c^2$$



# Types of Gears

‘Rack and pinion’ gears system looks unusual...composed of two gears. The ‘pinion’ is the normal round gear. The ‘rack’ is straight or flat. The rack has teeth cut in it and mesh with the teeth of the pinion gear.



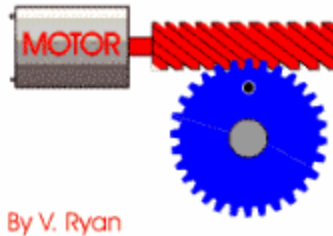
The pinion rotates and moves the rack in a straight line

- Another way of describing this is to say ‘rotary motion’ changes to ‘linear motion’.

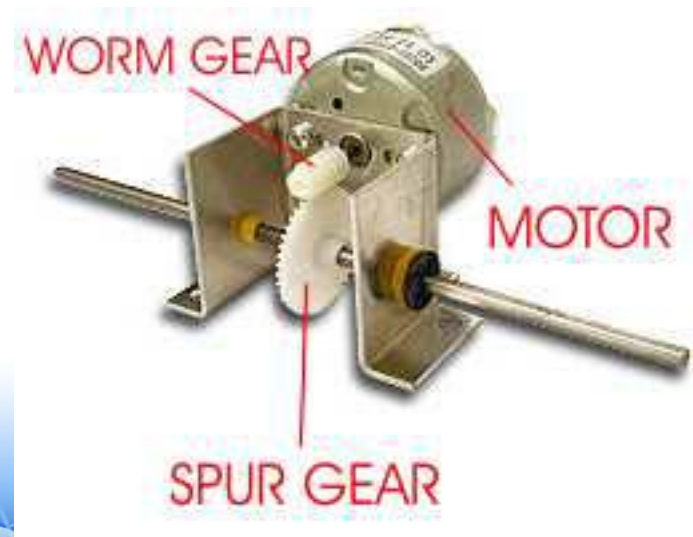
$$a^2 + b^2 = c^2$$



# Types of Gears

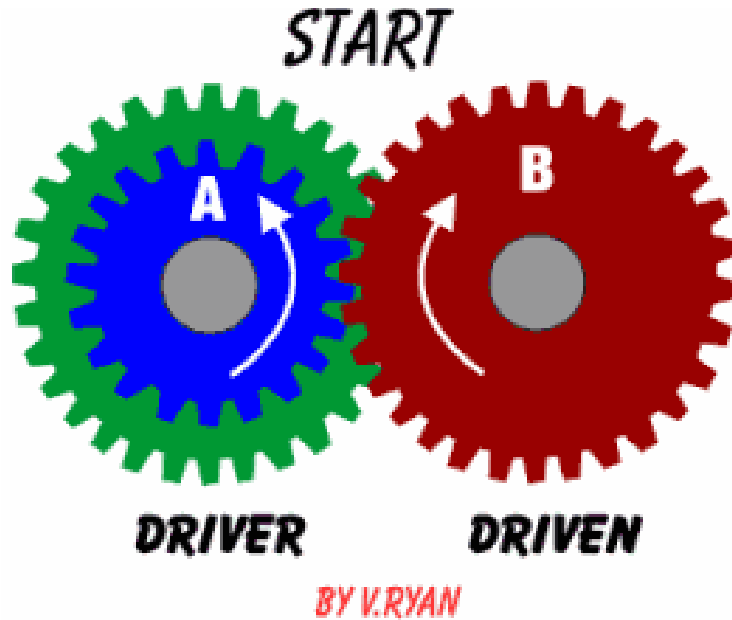


This arrangement of gears is called a *worm* and *wormwheel*. The worm is **brown** in color, only has **one** tooth but it is like a screw thread. The wormwheel, yellow, is like a normal gear wheel or spur gear. **The worm always drives the worm wheel round, ...never the opposite way as the system tends to lock and jam.**





# Gear Systems



**Compound gears** are used in engines, workshop machines and in many other mechanical devices.

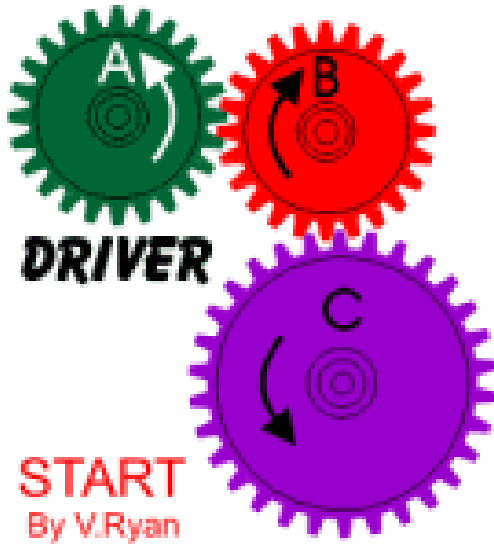
In the diagram, gear 'A' is actually 2 gears attached to each other. They rotate around the same centre.

Sometimes compound gears are used so that the final gear in a gear train rotates at the correct speed.

$$a^2 + b^2 = c^2$$



# Gear Systems



This is a good example of a ‘gear train’.

A gear train is usually made up of two or more gears.

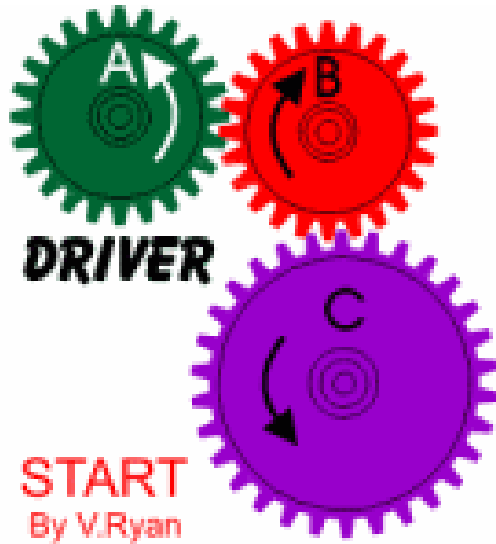
The driver in this example is gear ‘A’. If a motor turns gear ‘A’ in an anticlockwise direction;

Which direction does gear ‘B’ turn ?

$$a^2 + b^2 = c^2$$



# Gear Systems



This is a good example of a ‘gear train’.

A gear train is usually made up of two or more gears.

The driver in this example is gear ‘A’. If a motor turns gear ‘A’ in an anticlockwise direction;

Which direction does gear ‘B’ turn ?

**Clockwise**

$$a^2 + b^2 = c^2$$



# Gear Systems



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A gear train is usually made up of two or more gears.

The driver in this example is gear ‘A’. If a motor turns gear ‘A’ in an anticlockwise direction;

Which direction does gear ‘B’ turn ?

**Clockwise**

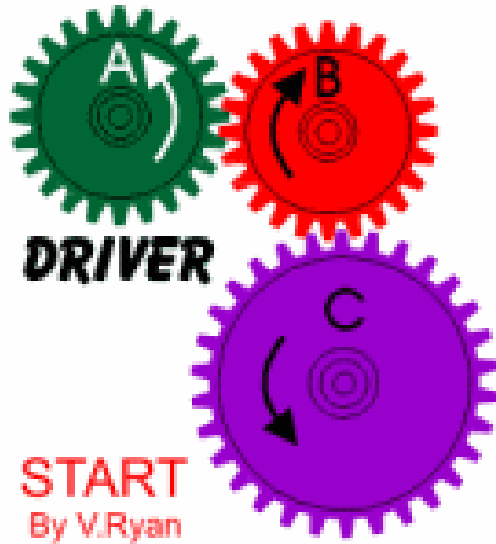
Which direction does gear ‘C’ turn ?

$$a^2 + b^2 = c^2$$





# Gear Systems



This is a good example of a ‘gear train’.

A gear train is usually made up of two or more gears.

The driver in this example is gear ‘A’. If a motor turns gear ‘A’ in an anticlockwise direction;

Which direction does gear ‘B’ turn ?

**Clockwise**

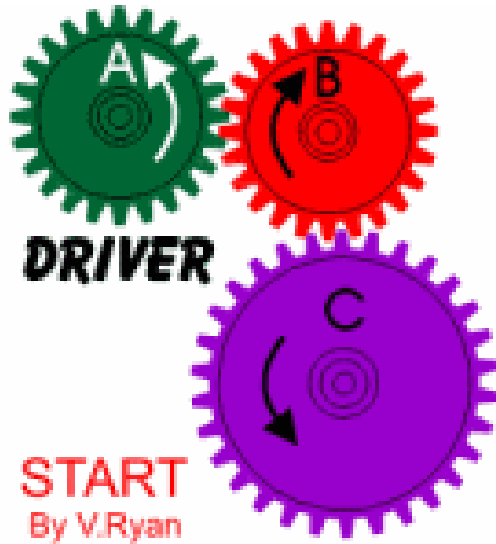
Which direction does gear ‘C’ turn ?

**Counter-Clockwise**

$$a^2 + b^2 = c^2$$



# Gear Systems



This is a good example of a ‘gear train’.

A gear train is usually made up of two or more gears.

The driver in this example is gear ‘A’. If a motor turns gear ‘A’ in an anticlockwise direction;

Which direction does gear ‘B’ turn ?

**Clockwise**

Which direction does gear ‘C’ turn ?

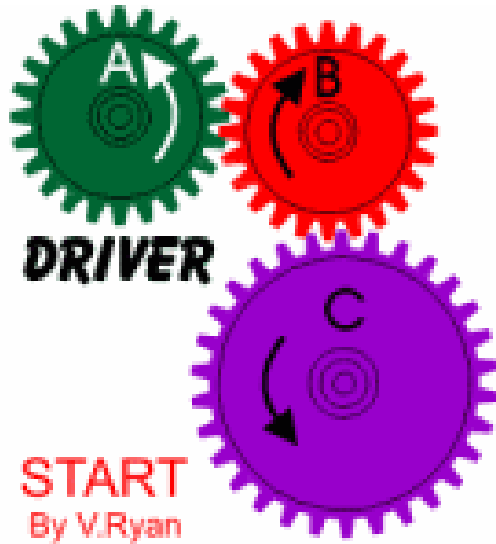
**Counter-Clockwise**

Does gear ‘C’ revolve faster or slower than gear ‘A’ ? - explain your answer.’

$$a^2 + b^2 = c^2$$



# Gear Systems



This is a good example of a ‘gear train’.

A gear train is usually made up of two or more gears.

The driver in this example is gear ‘A’. If a motor turns gear ‘A’ in an anticlockwise direction;

Which direction does gear ‘B’ turn ?

**Clockwise**

Which direction does gear ‘C’ turn ?

**Counter-Clockwise**

Does gear ‘C’ revolve faster or slower than gear ‘A’ ? - explain your answer.’

**SLOWER – SMALLER GEAR TURNS A LARGER GEAR**

$$a^2 + b^2 = c^2$$

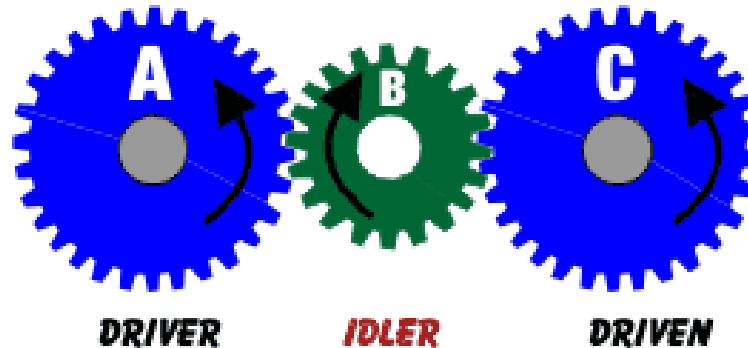


# Gear Systems

So far we have learned about **DRIVER** gears, **DRIVEN** gears and gear **TRAINS**. An **'IDLER'** gear is another important gear.

In the example opposite gear 'A' turns in an anticlockwise direction and also gear 'C' turns in an anticlockwise direction.

The 'idler' gear is used so that the rotation of the two important gears is the same.

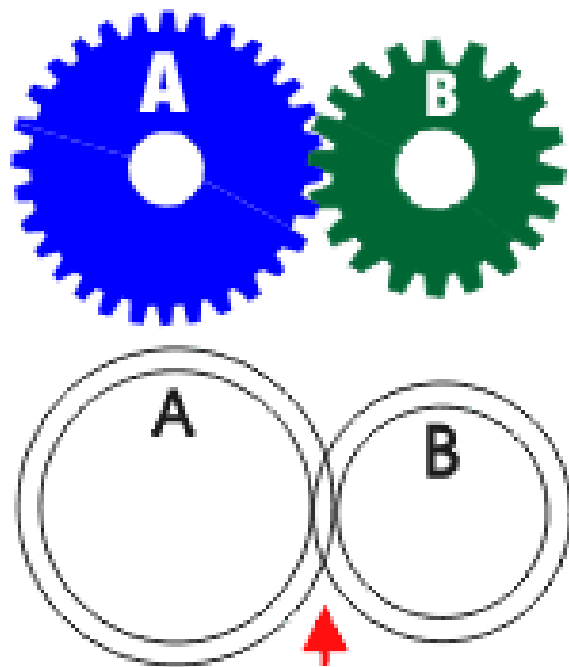


$$a^2 + b^2 = c^2$$





# *Drawing Gears*



**CIRCLES OVERLAP WHERE TEETH MESH**

It would be difficult to draw gears if you had to draw all the teeth every time you wanted to design a gear system.

For this reason a gear can be represented by drawing two circles.

$$a^2 + b^2 = c^2$$

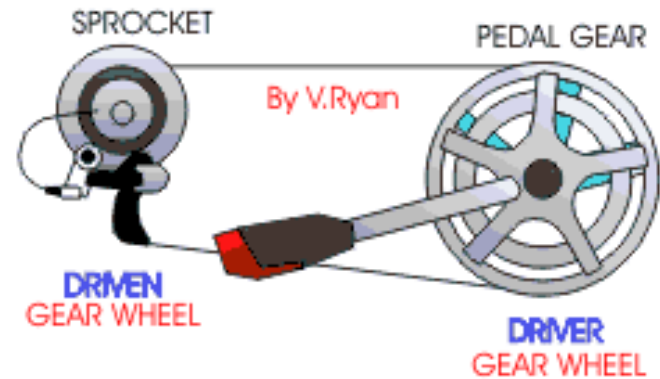
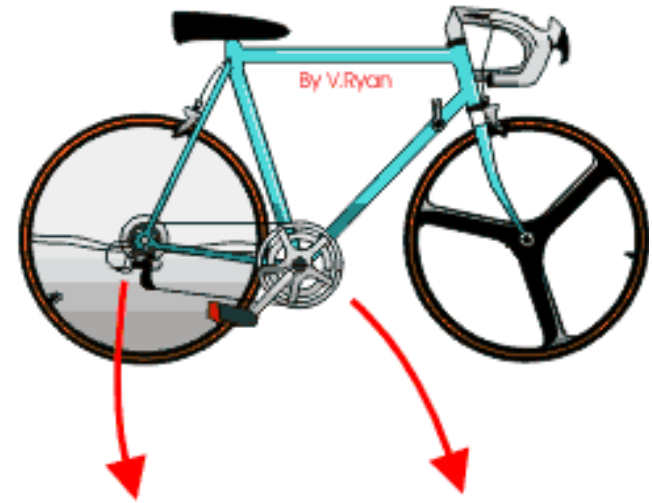


# *Gear Ratio(Velocity Ratio)*

Many machines use gears. A very good example is a bicycle which has gears that make it easier to cycle, especially up hills.

Bicycles normally have a large gear wheel which has a pedal attached and a selection of gear wheels of different sizes, on the back wheel.

When the pedal is revolved the chain pulls round the gear wheels at the back.

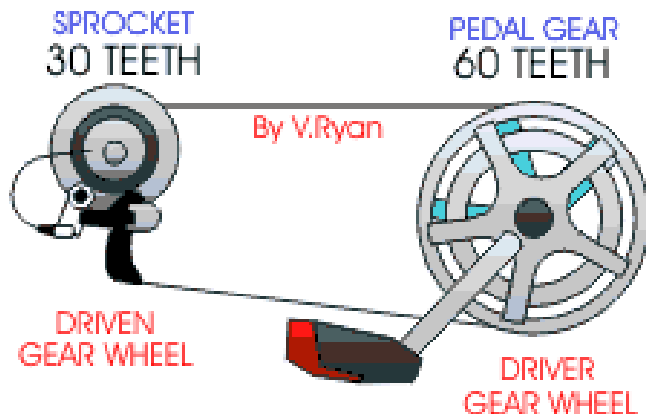


$$a^2 + b^2 = c^2$$



# Gear Ratio (Velocity Ratio)

The reason bicycles are easier to cycle up a hill when the gears are changed is due to what is called Gear Ratio (velocity ratio). Gear ratio can be worked out in the form of numbers and examples are shown. Basically, the ratio is determined by the number of teeth on each gear wheel, the chain is ignored and does not enter the equation.



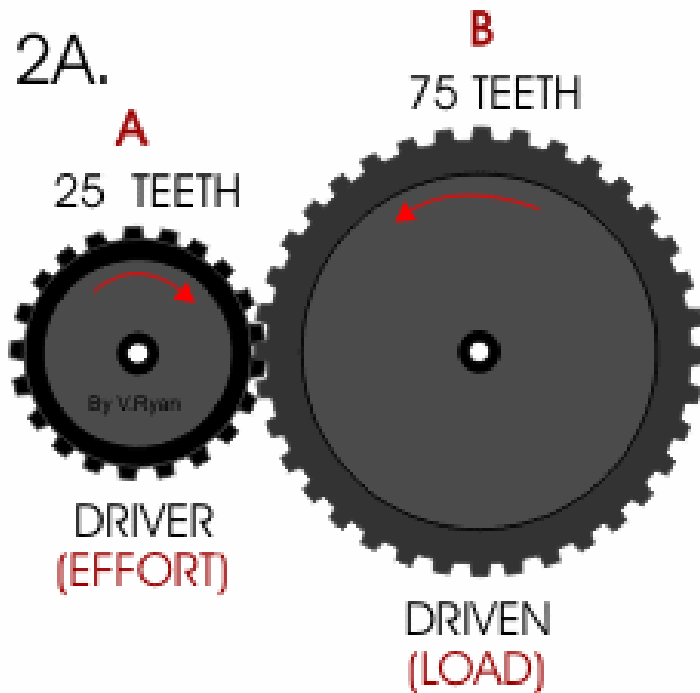
$$\frac{\text{Driven}}{\text{Driving}} = \frac{30}{60} = \frac{1}{2} \rightarrow 1:2$$

But **WHAT** does this mean? It means that the DRIVEN gear makes TWO rotations for every ONE rotation of the Driving Gear.

$a^2 + b^2 = c^2$



# Gear Ratio - Examples



$$\frac{\text{Driven}}{\text{Driving}} = \frac{75}{25} = \frac{3}{1} \rightarrow 3:1$$

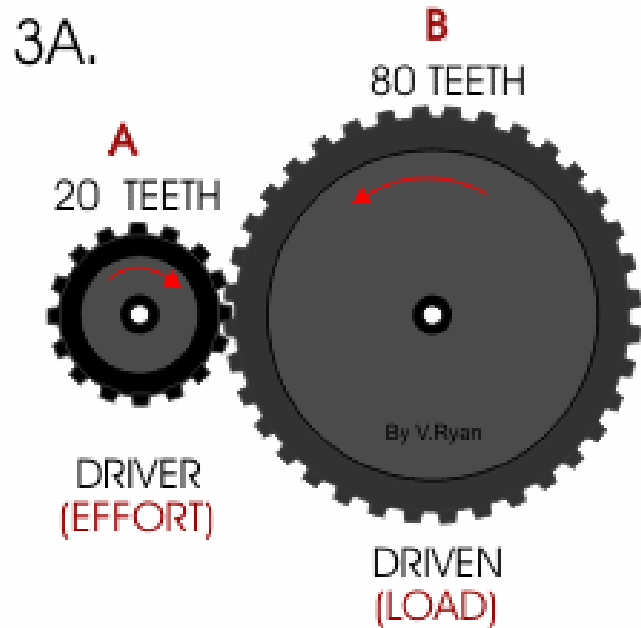
**What does this mean?** For every 3 rotations of the driving gear, the driven gear makes one rotation.

$$a^2 + b^2 = c^2$$





# Gear Ratio - Examples



$$\frac{\text{Driven}}{\text{Driving}} = \frac{80}{20} = \frac{4}{1} \rightarrow 4:1$$

**What does this mean?** For every 4 rotations of the driving gear, the driven gear makes 1 rotation.

$$a^2 + b^2 = c^2$$



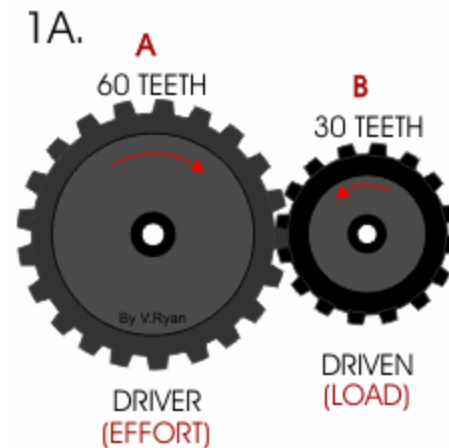
# Working out RPMs (revolutions per minute)

In the example shown, the DRIVER gear is larger than the DRIVEN gear.

The general rule is - large to small gear means 'multiply' the velocity ratio by the rpm of the first gear.

Divide 60 teeth by 30 teeth to find the velocity ratio(1:2).

Multiply this number (2) by the rpm (120). This gives an answer of 240rpm



GEAR A	GEAR B
60 teeth	30 teeth
120 rpm	?

$$\frac{60}{30} = 2$$
$$= 120 \times 2 = 240 \text{ revs/min}$$



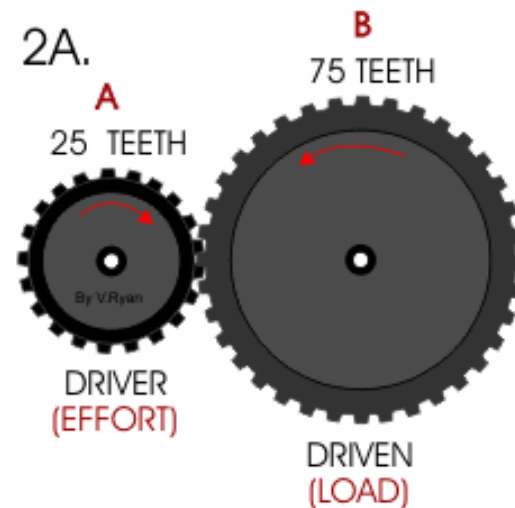
# Working out RPMs (revolutions per minute)

In the example shown, the DRIVER gear is smaller than the DRIVEN gear.

The general rule is - small to large gear means 'divide' the velocity ratio (3:1) by the rpm of the first gear.

Divide 75 teeth by 25 teeth to find the velocity ratio.  
divide the 60rpm by the velocity ration (3).

The answer is 20rpm.



GEAR A	GEAR B
25 teeth	75 teeth
60 rpm	?

$$\frac{75}{25} = 3$$
$$= \frac{60}{3} = 20 \text{ revs/min}$$

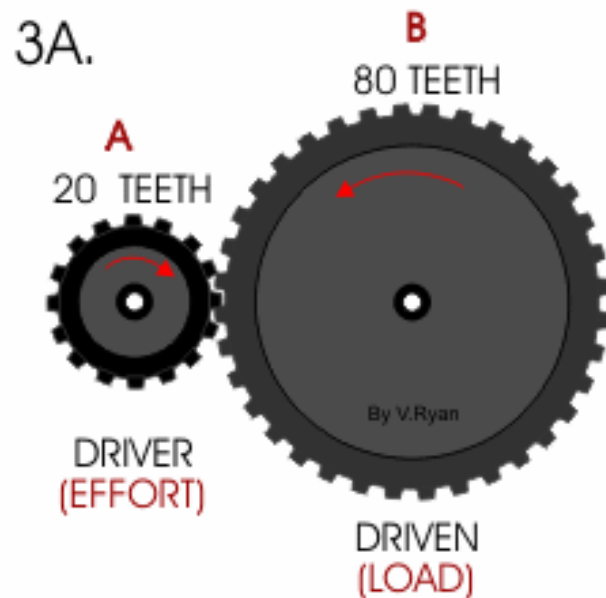
$$a^2 + b^2 = c^2$$



## Working out RPMs (revolutions per minute)

If A revolves at 100 revs/min what is B ?

(Remember small gear to large gear decreases revs)



GEAR A	GEAR B
20 teeth	80 teeth
100 rpm	?

$$\frac{80}{20} = 4$$
$$= \frac{100}{4} = 25 \text{ revs/min}$$

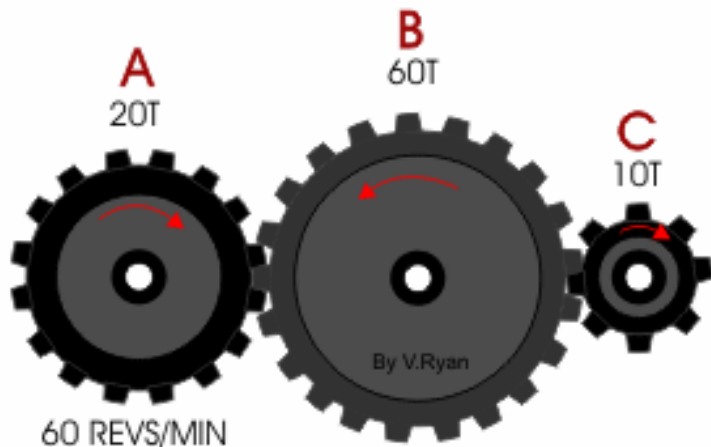
$$a^2 + b^2 = c^2$$





# Compound Gear Ratios

When faced with three gears the question can be broken down into two parts. First work on Gears *A* and *B*. When this has been solved work on gears *B* and *C*.



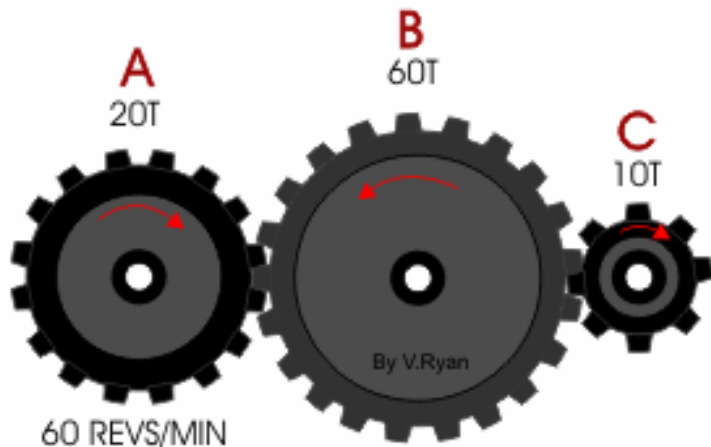
The diagram shows a gear train composed of three gears. Gear A revolves at 60 revs/min in a clockwise direction.

$$a^2 + b^2 = c^2$$



# Compound Gear Ratios

When faced with three gears the question can be broken down into two parts. First work on Gears *A* and *B*. When this has been solved work on gears *B* and *C*.



The diagram shows a gear train composed of three gears. Gear A revolves at 60 revs/min in a clockwise direction.

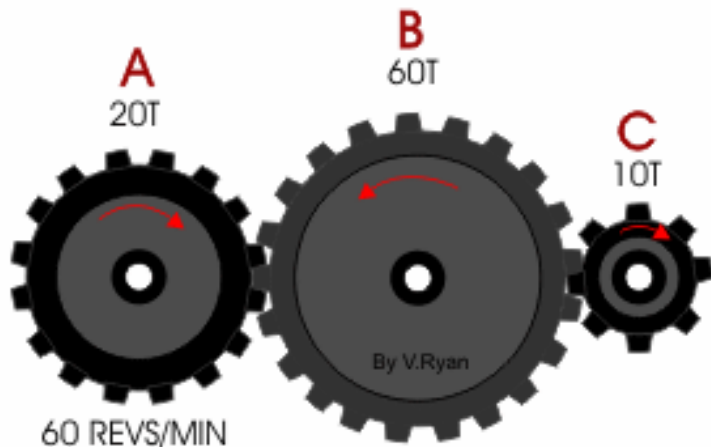
**What is the output in revolutions per minute at Gear C?**

$$a^2 + b^2 = c^2$$



# Compound Gear Ratios

When faced with three gears the question can be broken down into two parts. First work on Gears *A* and *B*. When this has been solved work on gears *B* and *C*.



The diagram shows a gear train composed of three gears. Gear A revolves at 60 revs/min in a clockwise direction.

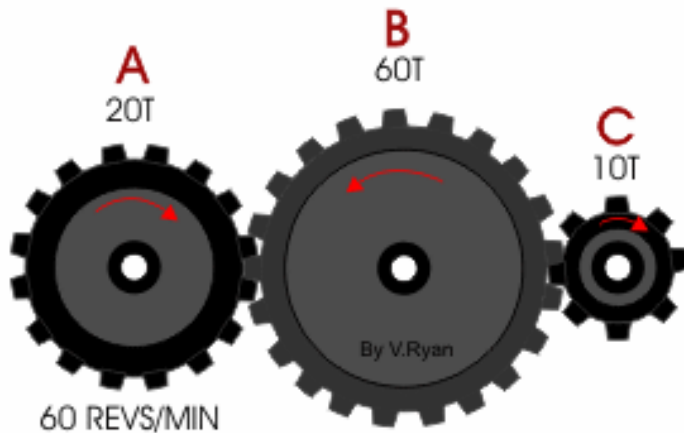
**What is the output in revolutions per minute at Gear C?**

**In what direction does Gear C revolve ?**

$$a^2 + b^2 = c^2$$



# Compound Gear Ratios



GEAR A	GEAR B	GEAR C
20 teeth	60 teeth	10 teeth

$$\frac{\text{Driven}}{\text{Driving}} = \frac{60}{20} = \frac{3}{1} \rightarrow 3:1$$

This means that for every THREE revolutions of GEAR A, Gear B travels once. Since we are going from a SMALLER gear to a LARGER gear we DIVIDE the Rpm's.

$$\frac{60 \text{ rev/min}}{3} = 20 \text{ rev/min}$$

Now find the gear ratio for B & C.

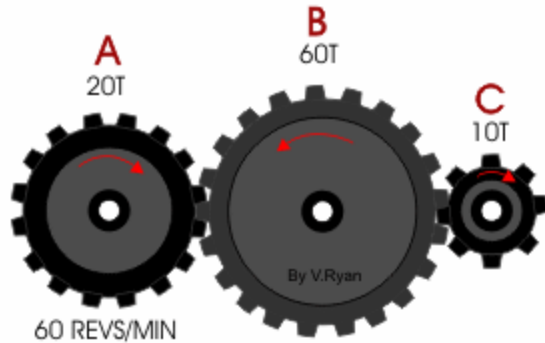
$$\frac{\text{Driven}}{\text{Driving}} = \frac{10}{60} = \frac{1}{6} \rightarrow 1:6$$

This means for every ONE rotation of gear B, gear C makes SIX rotations.

$$20 \text{ rev/min} \bullet 6 = 120 \text{ rev/min}$$



# Is there an easier way?



$$\frac{\text{Driven}}{\text{Driving}} = \frac{60}{20} \cdot \frac{10}{60} = \frac{10}{20} = \frac{1}{2} \rightarrow 1:2$$

You can also multiply the two gear ratios together to get the TOTAL gear ratio.

In the above figure we see that gear C will make TWO rotations for every one rotation of gear A.

Since gear C is smaller than gear A we multiply.

$$60 \text{ rev/min} \cdot 2 = 120 \text{ rev/min}$$

$$a^2 + b^2 = c^2$$



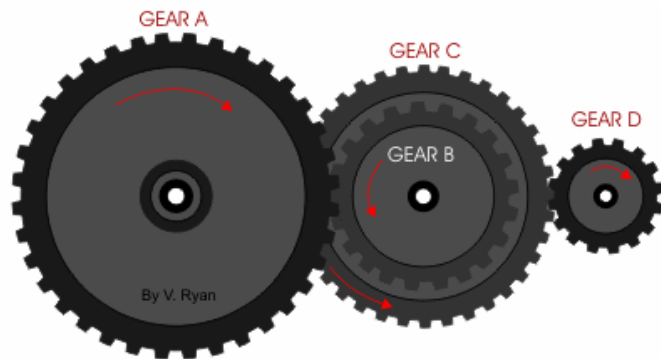


# Compound Gear Ratios

Below is a question regarding '*compound gears*'. Gears C and B represent a compound gear as they appear 'fixed' together.

When drawn with a compass they have the same center.

Two gears 'fixed' together in this way rotate together and at the same RPM. When answering a question like this split it into two parts. Treat gears **A** and **B** as one question *AND* **C** and **D** as the second part.



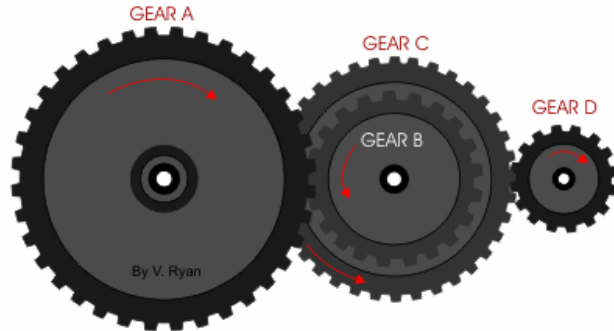
**What is the output in revs/min at D and what is the direction of rotation if Gear A rotates in a clockwise direction at 30 revs/min?**

$$a^2 + b^2 = c^2$$



# Compound Gear Ratios

Gear A	Gear B	Gear C	Gear D
120 teeth	40 teeth	80 teeth	20 teeth



$$\frac{\text{Driven}}{\text{Driving}} = \frac{40}{120} = \frac{1}{3}$$

$$\frac{\text{Driven}}{\text{Driving}} = \frac{20}{80} = \frac{1}{4}$$

Considering that Gear B is smaller than Gear A we can conclude that the RPMs for gear B is  $30 * 3 = 90$  rev/min

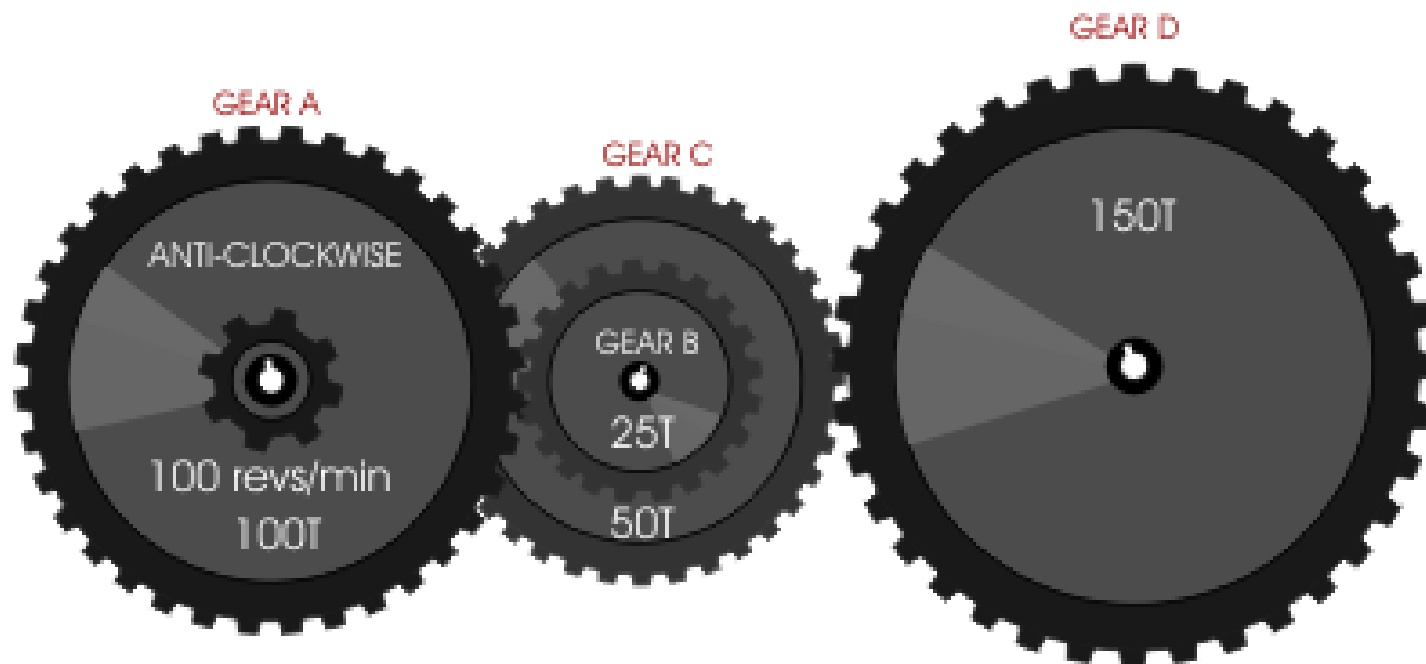
Since Gear B is at 90rev/min and has the SAME rotational speed as gear C Multiply by 4 to get Gear D's speed. Thus, Gear D moves at  $90 * 4 = 360$  rev/min

OR  $\frac{1}{3} \bullet \frac{1}{4} = \frac{1}{12}$

Since Gear A moves at 30rpms and Gear D is SMALLER. We multiply by 12.  $30 * 12 = 360$  rev/min



# Try this one

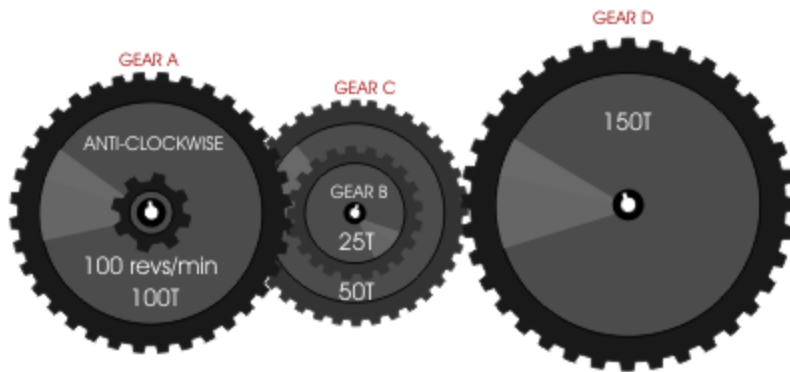


What is the revs/min at gear D and what is its direction?

$$a^2 + b^2 = c^2$$



# Answer



What is the revs/min at gear D and what is its direction?

If Gear A turns CCW, then gear B turns CW along with gear C as they are a compound gear. Therefore, **Gear D rotates CCW.**

Gear A	Gear B	Gear C	Gear D
100T	25T	50T	150T
100 rpm			

$$\frac{\text{Driven}}{\text{Driving}} = \frac{25}{100} = \frac{1}{4} \rightarrow 1:4$$

$$100 \cdot 4 = 400 \text{rpm} = \text{gear } b$$

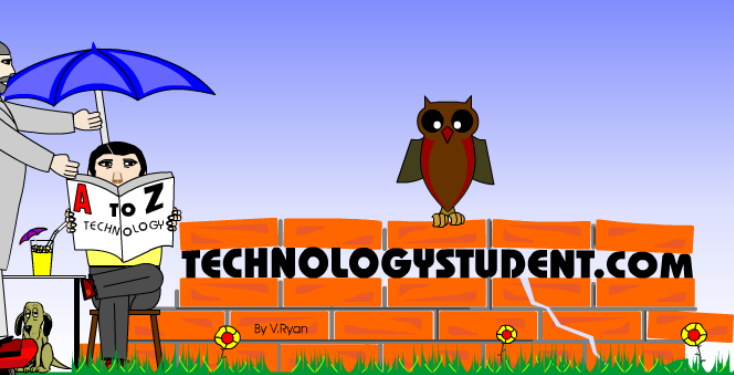
$$\frac{\text{Driven}}{\text{Driving}} = \frac{150}{50} = \frac{3}{1} \rightarrow 3:1$$

$$\frac{400}{3} = 133.3 \text{rpm} = \text{gear } D$$

$$a^2 + b^2 = c^2$$







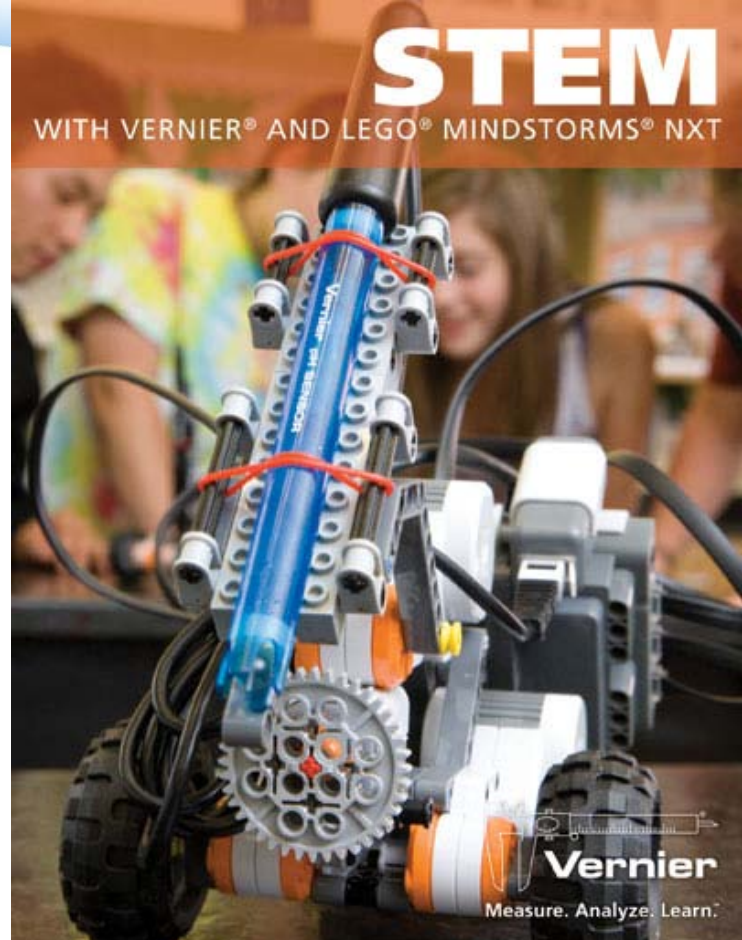
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By V.Ryan

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DESIGN PROCESS CONCISE VERSION	STRUCTURES	FORCES / MOMENTS
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ELECTRONICS	ROBOLAB	COMPUTER CONTROL
MECHANISMS	GEARS AND PULLEYS	TECHNOLOGY AND CULTURES
HEALTH AND SAFETY	RESISTANT MATERIALS	EQUIPMENT AND PROCESSES
TECHNOLOGY AND THE ENVIRONMENT	CNC WORK	DIPLOMA/ VOCATIONAL WORK
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## Resources



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Education NXT Base Set  
(W979797)

**LEGO® MINDSTORMS®**  
Education NXT Software  
2.0 (W900080)

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NXT The Mayan  
Adventure (W991341)



$$a^2 + b^2 = c^2$$

Math Connections to Earth and Space Science

